Effect of Aperture Averaging Upon Tropospheric

Phase Fluctuations Seen With a Radio Antenna

by

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ABSTRACT

The spectrum of tropospheric phase fluctuations expected for a radio antenna at timescales < 100 s on a space to ground link has been calculated. A new feature included in these calculations is the effect of aperture averaging, which causes a reduction in delay fluctuations on timescales less than the antenna windspeed crossing time, $\sim D/(8 \text{ m/s})$ (D is the antenna diameter). On timescales less than a few seconds, the Allan Deviation $\sigma_y(\Delta t) \propto (\Delta t)^{+1}$, rather than $\sigma_y(\Delta t) \propto (\Delta t)^{-1/6}$ without aperture averaging. Calibration of tropospheric phase fluctuations with Water Vapor Radiometers will not be possible on timescales less than ≈ 10 s, due to thermal radiometer noise. However, the tropospheric fluctuation level will be small enough that phase measurements on timescales less than a few seconds will be limited by the stability of frequency standards and/or other nontropospheric effects.

I. INTRODUCTION

The earth's neutral atmosphere limits the precision of phase measurements with microwave signals transmitted between earth and space, or between two ground antennas. The atmospheric refractivity is a known function of the pressure, temperature, and relative humidity. However, fluctuations in these properties cause time variability in the electrical path length.

There are a number of potential motivations for precise phase measurements of microwave transmissions. One particular application involves spacecraft sent to other planets. For spacecraft beyond the earth-moon system, the radio link between a ground antenna and the spacecraft is often used for 'radio science' experiments (*e.g.* Tyler *et al.*, 1989). The received amplitude and phase are used to measure (or search for) quantities such as gravitational radiation, refractivity profiles of the atmosphere of the planet or planetary satellite near the spacecraft, or the structure of a planetary ring system. Any perturbations on the link that are caused by the media between the ground antenna and the spacecraft will corrupt the accuracy of the radio science measurement (unless the purpose of the experiment is to study that medium, *e.g.* the solar plasma). With the trend towards higher RF frequency for spacecraft-ground links, the relative magnitude of tropospheric and charged particle (earth ionosphere and interplanetary medium) phase fluctuations has changed dramatically. This is because plasma is dispersive at microwave frequencies, and the troposphere is not. At S-band (2.3 GHz), charged particle phase fluctuations dominate (especially for spacecraft beyond earth orbit, where the interplanetary medium contributes). At Ka-band (32 GHz), tropospheric phase fluctuations will dominate except at small sun-spacecraft angular separations. The Cassini spacecraft, launched in October, 1997 for a mission to Saturn, is the first planetary spacecraft with a two way Ka-band link capability, and ambitious radio science measurements are planned (Comoretto *et al.*, 1992; Rappaport *et al.*, 1997; Kliore *et al.*, 1998).

Knowledge of the tropospheric fluctuation spectrum will be useful in planning and analyzing these radio science experiments. Comparison of fluctuation levels measured by Very Long Baseline Interferometry (VLBI) and Water Vapor Radiometers (WVRs) has demonstrated that tropospheric delay/phase fluctuations at microwave frequencies are dominated by fluctuations in water vapor density (Elgered *et al.*, 1991). The spectrum of tropospheric fluctuations has been measured by VLBI on timescales > 20 s (Linfield *et al.*, 1996) and by WVRs on timescales ≥ 200 s (Keihm, 1995). A model for these fluctuations has been developed (Treuhaft and Lanyi, 1987). Previous calculations using this model have generally assumed a zero thickness 'pencil beam' for the ground radio antenna. However, on timescales less than ≈ 10 s, the nonzero diameter of this antenna will modify the pencil beam spectrum. The method for calculating this 'aperture averaging' effect is presented in Section II, and results are given in Section III. A short summary is given in Section IV.

II. CALCULATION OF APERTURE AVERAGING

Both theoretical arguments (Tatarski, 1961) and observational data (Rogers *et al.*, 1984; Treuhaft and Lanyi, 1987; Linfield *et al.*, 1996) support the idea that tropospheric refractivity fluctuations obey a Kolmogorov power law. The refractivity structure function $D_n(r)$ is:

$$D_n(r) \equiv \left\langle [N(\vec{x}+\vec{r}) - N(\vec{x})]^2 \right\rangle = C_n^2 r^{2/3} \qquad (1)$$

$N(\vec{x})$ is the refractivity at location $\vec{x}$ ($N \equiv n-1$, where n is the index of refraction), and C_n is the refractivity structure constant. At optical frequencies, refractivity fluctuations are due mainly to temperature/density variations of dry air. When the atmosphere is stable with respect to convection, the outer scale size for these varations is typically $\sim$ 10 m (Coulman *et al.*, 1988; Ziad *et al.*, 1994). However, at radio frequencies, the refractivity of water vapor is much higher (Thayer, 1974), and refractivity fluctuations are dominated by variations in the vapor density. The outer scale size in the structure functions (eq. 1) for these vapor density variations is typically tens of km or larger.

Over timescales up to thousands of seconds, time variations in line-of-sight tropospheric delay can be successfully represented by a 'frozen flow' model, in which spatial variations are convected past the observer by the wind. Because the tropospheric delay is nearly nondispersive at microwave frequencies, the phase fluctuations will be linearly proportional to the RF frequency.

The height dependence of C_n is poorly constrained by data. It was modeled by (Treuhaft and Lanyi, 1987) as constant up to a 1 km height, and zero above. Based on additional data and analysis, this 'slab height' was subsequently revised to 2 km (R. Treuhaft, private communications). Recent WVR data (Keihm, 1995) from Goldstone, California (one of the three complexes in the Deep Space Network of the National Aeronautics and Space

Administration) show that the observed fluctutations agree with the predictions of this 2 km slab model on timescales > 400 s, but are lower than model predictions on shorter timescales (*i.e.* the measured spectrum is flatter than the model spectrum). This result is most easily explained with a thickness of the turbulent layer which is significantly larger than 2 km, although the data are not adequate to solve for a specific value for this thickness. The effect of a finite thickness h to the medium is that fluctuations in the vertical dimension saturate on timescales $> h/v_w$, where v_w is the wind speed. For timescales $\Delta t << h/v_w$, the delay structure function $D_\tau \propto (\Delta t)^{5/3}$, and for timescales $\Delta t >> h/v_w$, $D_\tau \propto (\Delta t)^{2/3}$ (Treuhaft and Lanyi, 1987). The timescales covered in this report are all $<< h/v_w$. WVR measurements of fluctuations on timescales of 200 s (Keihm, 1995) were used to derive the mean C_n, once h was chosen. For $h = 4$ km, $C_n = 3.0 \times 10^{-8}$ m$^{-1/3}$; these values were used for all calculations presented in this report. Over the course of a year, the structure constant at Goldstone exhibits variations of at least a factor of 2 about its mean value (Keihm, 1995).

The tropospheric delay for a pencil beam looking from location $\vec{x}$ at elevation angle θ and azimuth AZ is:

$$\tau_{pencil}(\vec{x}, \theta, AZ) = \frac{1}{\sin\theta} \int_0^\infty N\left[\vec{x} + \vec{r}(\theta, AZ, z)\right] dz \tag{2}$$

$\vec{r}(\theta, AZ, z)$ is the vector from the surface to height z, in direction (θ, AZ). For a radio antenna of diameter d, the delay is averaged over the circular aperture, to give:

$$\tau_{antenna}(\vec{x}, \theta, AZ) = \frac{4}{\pi d^2} \int_0^{2\pi} \int_0^{d/2} \xi \tau_{pencil}\left[\vec{x} + \vec{r}(\xi, \phi), \theta, AZ\right] d\xi d\phi \tag{3}$$

$\vec{r}(\xi, \phi)$ is the vector in the plane of the aperture, starting from the center, of length ξ and at angle ϕ (the direction chosen for $\phi = 0$ does not affect the result). Eq.(3) assumes that the near field antenna beam profile is uniform across its circular cross section. The true profile will be tapered towards the edges. As presented in Section III, a change of a factor of two in antenna diameter causes a change in the resulting fluctuation level by a factor of ≈ 2. Therefore, neglect of beam tapering should cause an error of $< 20\%$ in the results reported here.

A convenient way of calculating and expressing the tropospheric fluctuation spectrum is Allan Variance $\sigma_y^2(\Delta t)$, defined for a delay process $\tau(t)$ as (Allan, 1996):

$$\sigma_y^2(\Delta t) \equiv \frac{\left\langle [\tau(t+2\Delta t) - 2\tau(t+\Delta t) + \tau(t)]^2 \right\rangle}{2\,(\Delta t)^2} \tag{4}$$

Expanding eq.(4) and assuming that averaged quantities are independent of time and position gives, for the tropospheric delay fluctuations measured by a radio antenna:

$$\begin{aligned} \sigma_y^2(\Delta t) =& \frac{3\left\langle \tau_{antenna}^2(t) \right\rangle}{(\Delta t)^2} - \frac{4\left\langle \tau_{antenna}(t+\Delta t)\tau_{antenna}(t) \right\rangle}{(\Delta t)^2} \\ &+ \frac{\left\langle \tau_{antenna}(t+2\Delta t)\tau_{antenna}(t) \right\rangle}{(\Delta t)^2} \end{aligned} \tag{5}$$

We can evaluate this expression with eqs. (3) and the frozen flow assumption. Taking the middle term in eq.(5) as an example,

$$\begin{aligned} \left\langle \tau_{antenna}(t+\Delta t)\tau_{antenna}(t) \right\rangle &= \frac{16}{\pi^2 d^4} \int_0^{2\pi} \int_0^{2\pi} \int_0^{d/2} \\ \cdot \int_0^{d/2} \xi\xi' \left\langle \tau_{pencil}\left[\vec{x} + \vec{v}_w \Delta t + \vec{r}(\xi,\phi)\right] \tau_{pencil}\left[\vec{x} + \vec{r}(\xi',\phi')\right] \right\rangle d\xi d\xi' d\phi d\phi' \end{aligned}$$

It has been assumed that averaged quantities are position independent, so that

$$\begin{aligned} &\left\langle \tau_{pencil}(\vec{x}_1,\theta,AZ)\tau_{pencil}(\vec{x}_2,\theta,AZ) \right\rangle \\ &\quad = \left\langle \tau_{pencil}^2(\vec{x},\theta,AZ) \right\rangle - \frac{1}{2}\left\langle [\tau_{pencil}(\vec{x}_1,\theta,AZ) - \tau_{pencil}(\vec{x}_2,\theta,AZ)]^2 \right\rangle \\ &\quad = \left\langle \tau_{pencil}^2(\vec{x},\theta,AZ) \right\rangle - \frac{1}{2} D_\tau\left(|\vec{x}_1 - \vec{x}_2|\right) \end{aligned}$$

D_τ is the pencil beam delay structure function, defined as:

$$D_\tau(r) \equiv \left\langle [\tau_{pencil}(\vec{x}+\vec{r},\theta,AZ) - \tau_{pencil}(\vec{x},\theta,AZ)]^2 \right\rangle$$

In evaluating eq.(5), the $\left\langle \tau^2_{pencil}(\vec{x}) \right\rangle$ terms add to zero, and $\sigma_y^2(\Delta t)$ can be expressed as a four-dimensional integral of three D_τ terms:

$$\begin{aligned}
\sigma_y^2(\Delta t) =& \frac{16}{\pi^2 d^4 (\Delta t)^2} \int_0^{2\pi} d\phi \int_0^{2\pi} d\phi' \int_0^{d/2} \xi d\xi \\
&\cdot \int_0^{d/2} \xi' d\xi \left[-\frac{3}{2} D_\tau(A) + 2D_\tau(B) - \frac{1}{2} D_\tau(C) \right] \\
A \equiv& |\vec{r}(\xi,\phi) - \vec{r}(\xi',\phi')| \\
B \equiv& |\vec{v}_w \Delta t + \vec{r}(\xi,\phi) - \vec{r}(\xi',\phi')| \\
C \equiv& |2\vec{v}_w \Delta t + \vec{r}(\xi,\phi) - \vec{r}(\xi',\phi')|
\end{aligned} \tag{6}$$

The horizontal distance scales are d and $v_w \Delta t$. As a result, $\sigma_y(\Delta t)$ will be suppressed, relative to its $d = 0$ value, for values of Δt smaller than a few times the windspeed crossing time of $t_{cross} = d/v_w$. For a typical wind speed in the lower few km of 8 m/s and for $d = 34$ m (the most common antenna size at the Goldstone site), $t_{cross} \approx 5$ s. For $\Delta t << t_{cross}$, $\sigma_y(\Delta t)$ will be much lower than its $d = 0$ value, due to the averaging of many small scale delay fluctuations over the much larger antenna aperture. An alternate treatment of this problem, using Power Spectral Density instead of Allan Variance, and emphasizing the coherence of interferometric observations, can be found in (Lay, 1997).

The time variations as characterized above will be modified by motion of the antenna. For a spacecraft moving through the sky at an angular rate Ω, the tracking will cause an effective velocity v_{eff} through the troposphere, at height z and elevation angle θ of:

$$v_{eff} = \frac{\Omega z}{\sin\theta}$$

If v_{eff} is smaller than 8 m/s, this angular motion will not make major changes to the calculation of $\sigma_y^2(\Delta t)$ presented above. This condition translates to

$$\Omega < \frac{8 \text{ m/s } \sin\theta}{z}$$

For $z = 2$ km (half the turbulent slab height),

$$\Omega < 0.004 \sin\theta = 0.2 \text{ degrees/s } \sin\theta$$

This condition is clearly satisfied for any spacecraft beyond earth orbit. For a close pass of an earth orbiter the altitude will be $\approx r \sin\theta$, where r is the range to the spacecraft. With an orbital velocity $\sim$ 10 km/s for the spacecraft, the above condition on Ω will be satisfied for spacecraft altitudes $>$ 2500 km. For altitudes $<$ 2500 km, the spacecraft motion can cause (depending on the geometry) an effective velocity which is comparable to, or larger than, the actual wind speed. In this case, the delay fluctuations will be *larger* than those presented here.

The Fresnel length scale l for observations at a wavelength λ, with phase perturbations occuring at a distance L, is $l \approx \sqrt{\lambda L}$. For $\lambda = 1$ cm and $L = 2$ km (the middle of the turbulent troposphere), $l \approx 4$ m. On smaller scales, geometric optics is not completely valid. The results for timescales $< 4 \text{ m/s}/v_w = 0.5$ s should therefore be regarded as approximate. The effect of diffraction is to move power to lower frequencies, so the qualitative conclusions in this report should stand.

III. RESULTS

Eq. (6) was evaluated numerically, for a range of time intervals, and for both 34 m and 70 m diameter antennas. A wind speed of 8 m/s was assumed. The results, for antennas pointed in the zenith direction, are shown in Figure 1. For comparison, $\sigma_y(\Delta t)$ for a pencil beam antenna is also shown.

For time intervals shorter than 10 s, the Allan Deviation for measurements made with a radio antenna are less than those made with a pencil beam. For time intervals less than ≈ 2 s, $\sigma_y(\Delta t) \propto (\Delta t)^k$, where $k \approx +1$. This proportionality can be roughly understood as follows. The effect of fluctuations over a time interval Δt are confined to a region of size $\approx v_w \Delta t$. For $v_w \Delta t << d$, there are $N_{regions} \sim (d/(v_w \Delta t))^2$ such regions over the antenna aperture. Averaging the delay fluctations of these regions will cause a reduction by a factor of $\sim \sqrt{N_{regions}} \sim d/(v_w \Delta t)$. The slope of the pencil beam $\sigma_y(\Delta t)$ is $\approx -1/6$, so the slope of $\sigma_y(\Delta t)$ as seen by an antenna of nonzero size will be $\approx +5/6$.

As Δt decreases, the amount of computer time needed to calculate $\sigma_y(\Delta t)$ increases rapidly. For small Δt, the terms A, B, and C in eq.(6) are nearly equal, and high numerical precision is needed in order to determine the differences in the D_τ terms. All derivatives of D_n and all but the first derivative of D_τ become singular at zero separation, so that numerical integration methods have difficulty.

Calculations for Δt as small as 0.02 s were possible in the zenith direction, because $D_\tau(r)$ could be evaluated separately, with an analytical formula used in eq.(6) to reduce the number of dimensions in the integration from 6 to 4. Calculations at other elevation angles were limited to a narrower range of time intervals (1–100 s). These calculations showed that $\sigma_y(\Delta t)$ increases slowly with decreasing elevation angle. Relative to its value at the zenith, $\sigma_y(\Delta t)$ was 1.16 times larger at 30° elevation angle, and 1.82 times larger at 10° elevation angle.

All $\sigma_y(\Delta t)$ values in Figure 1 are linearly proportional to the structure constant, C_n. Therefore, the actual $\sigma_y(\Delta t)$ values at Goldstone will range up and down from those in Figure 1 by a factor of at least 2, depending upon season, time of day, and weather conditions. Winter nights tend to have the lowest C_n values, with summer days having the largest values. Data from a global ground network of GPS receivers indicates that the mean value of C_n at Goldstone is a factor of ~ 2 smaller than at a typical ground location (C. Naudet, private communication).

Tropospheric delay fluctuations at microwave frequencies can be calibrated by Water Vapor Radiometers (WVRs), which measure thermal emission from water vapor in the vicinity of its 22 GHz spectral line (Elgered, 1993). However, on short timescales, the thermal noise from a WVR becomes the limiting error source. The main vapor-sensing channel of current WVRs is at a frequency near 21.5 GHz or 23.8 GHz, where 1K brightness temperature corresponds to ≈ 6 mm$/c = 2 \times 10^{-11}$ s of path delay (c is the speed of light). The thermal path delay equivalent noise in a WVR measurement of integration time t_{int} for a total system temperature T_{sys} and bandwidth BW is:

(7) $$N(t_{int}) = \frac{T_{sys}}{\sqrt{BW\, t_{int}}} \cdot \frac{2 \times 10^{-11}\ \text{s}}{1\text{K}} = \frac{2 \times 10^{-13} T_{100}}{BW_{100}^{0.5}\, t_{int_s}^{0.5}}\ \text{s}$$

$T_{100} \equiv T_{sys}/100\text{K}$, $BW_{100} \equiv BW/100$ MHz, and $t_{int_s} \equiv t_{int}/1$ s.

To set a firm lower limit on the effect of WVR thermal noise, we assume that $\Delta t = t_{int}$ (*i.e.* you give up all information on timescales shorter than Δt). Future uncooled WVRs currently under design at the Jet Propulsion Laboratory are expected to have $T_{sys} \approx 300$ K and $BW \approx 400$ MHz (A. B. Tanner, private communications). The Allan Deviation from thermal WVR noise is then, for $\Delta t_s = \Delta t/1$ s (note that $\sigma_y(\Delta t) = N(t_{int})\sqrt{3}/\Delta t$ for thermal noise)

(8) $$\sigma_y(\Delta t) \approx \frac{5 \times 10^{-13}}{\Delta t_s^{1.5}} \quad \text{(WVR Thermal Noise)}$$

This WVR thermal noise curve is plotted on Figure 1. If we require $\sigma_y(\Delta t)$ from WVR thermal noise to be at least 3 times smaller than the tropospheric $\sigma_y(\Delta t)$ in order to achieve useful calibration, then, for any elevation angle $\geq 10°$, the minimum time inteval Δt_{min} for WVR calibration is:

$$\Delta t_{min} \approx 10 \text{ s}$$

Cooled receivers would improve the sensitivity of WVRs. Sensing fluctuations at frequencies > 200 GHz, where the opacity of water vapor is larger than at 22 GHz, would also give better sensitivity. However, large radio antennas (such as those at Goldstone) do not have a surface usable at 200 GHz. The use of a smaller (and more precise) antenna for vapor sensing would introduce a mismatch between the volumes sensed with the main antenna beam and the calibration beam (Linfield and Wilcox, 1993). Because the difference in slopes of the tropospheric $\sigma_y(\Delta t)$ at $\Delta t \sim 10$ s and the WVR thermal noise $\sigma_y(\Delta t)$ is large, the minimum useful calibration interval is relatively insensitive to the values of T_{sys} and BW. *On timescales less than ≈ 10 s, calibration of the tropospheric fluctuations seen by a large radio antenna will be very difficult.*

IV. SUMMARY

The effect of aperture averaging upon radio science measurements is that tropospheric fluctuations will not be important on short timescales. For occultation measurements of Saturn's rings with the Cassini spacecraft, the shortest timescale of interest is the desired physical resolution (≈ 100 m) divided by the spacecraft orbital velocity (≈ 10 km/s), or ~ 0.01 s. These measurements will use an on-board oscillator with $\sigma_y(\Delta t) \geq 10^{-13}$. Even if future flight oscillators were a factor of 10 more stable than the one on Cassini, the troposphere would be more stable than the oscillator for $\Delta t < 1$ s. The tropospheric fluctuations on $\Delta t < 10$ s appear to form a fundamental lower limit for radio science measurements using an earth antenna, because WVR thermal noise precludes their calibration.

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FIGURE CAPTION

Figure 1

The Allan Deviation calculated for a pencil beam antenna and for antenna diameters of 34 and 70 m is shown as a function of time interval. These curves are for typical conditions at Goldstone, California, and for the zenith direction. The Allan Deviation at other elevation angles will be somewhat larger, increasing by a factor of 1.8 at 10° elevation angle. The thermal noise from a WVR with the characteristics expected for the calibration system currently under development at JPL is plotted as a dashed line.

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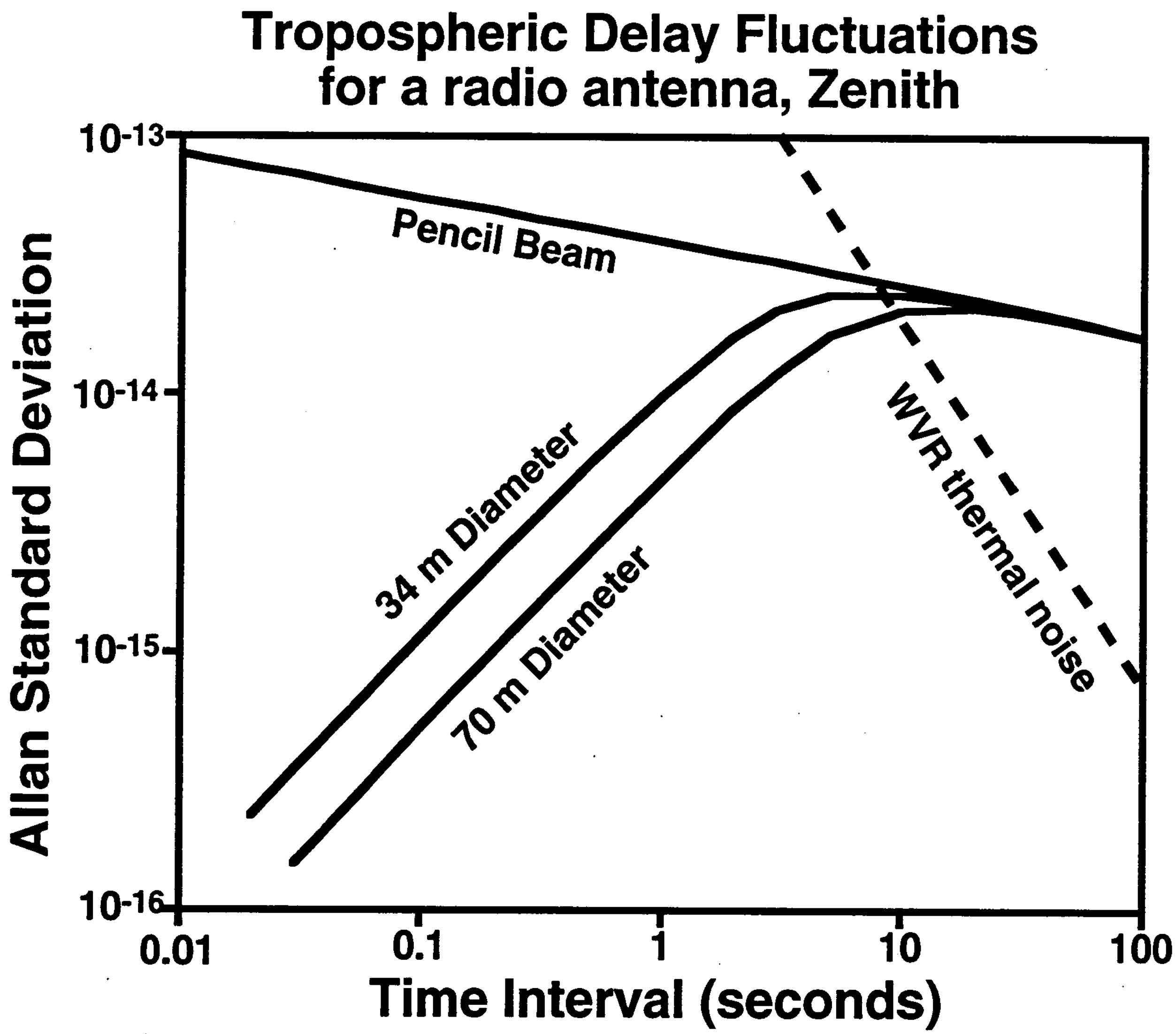

Tropospheric Delay Fluctuations
for a radio antenna, Zenith
Allan Standard Deviation
10-13
10-14
10-15
10-16
0.01
0.1
1
10
100
Time Interval (seconds)
Pencil Beam
34 m Diameter
70 m Diameter
WVR thermal noise